ANALYTICAL-NUMERICAL MODELING OF FLEXIBLE PYLONS FOR STRUCTURAL HEALTH MONITORING

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Abstract. The monitoring of flexible structures such as towers, antennas and pylons comprising critical infrastructure for power transmission and telecommunications is necessary in order to ensure their continuous operation. This particular field of engineering comes under the umbrella of structural health monitoring, which in recent years has been given high priority in developed countries. There, the built infrastructure is both extensive, encompassing diverse groups of systems such as energy, telecommunication, utility and transportation networks, while at the same time is ageing rapidly. The goal is to set up monitoring schemes with a stand-alone configuration and a wireless transmission of data to central locations for further processing. Thus, it becomes necessary for the sensors employed to have some structural analysis capabilities that can be used in tandem with their data processing functions, all within an artificial intelligence environment. To this end, we develop analytical models for vibrations in elastic waveguides that model flexible structures of variable cross-section and demonstrate their efficiency and accuracy through comparisons with finite element models.
1 INTRODUCTION

Broadly speaking, infrastructure can be classified as the built environment plus the necessary networks for the supply of energy, water, communications and transportation. Since network operation must remain continuous, Structural Health Monitoring (SHM) is required as a necessary step in setting up maintenance protocols [1]. The data streams generated by monitoring must be evaluated with the aid of Artificial Intelligence (AI) algorithms that will allow authorities to reach rational conclusions regarding the current state of network operation and to assess the need for maintenance, repair, retrofit and rehabilitation [2]. Within this cycle, it is necessary to use numerical models to analyze the current state of the structure as needed [3]. The usual path followed nowadays for modelling either the structure or the structural network under consideration is the Finite Element Method (FEM). This usually requires the setup of meshes for discretizing the continuum that are quite detailed and possibly cannot be accommodated within wireless sensor nodes with limited computational resources and power supply. It then becomes necessary to introduce analytical models with closed-form solutions [4] which can be programmed using computer platforms such as Java [5]. This representation is both accurate and efficient and requires small amounts of computing power. This way, the data streams generated by ambient vibrations and other external loads can be processed and compared with computed response markers [6, 7]. Then, these data streams will be either discarded or transmitted to a central processing unit for further evaluation. The final goal is to produce reliable information through SHM to decide if the structure in question is in need of repairs.

Flexible structures such as antennas and pylons used in power transmission and in telecommunications can be efficiently modelled as elastic waveguides, i.e. base-supported beams of variable cross-section with a continuous mass distribution. In general, these waveguides undergo axial, flexural and torsional vibrations, see Fig. 1. Their motion is governed by partial differential equations in a space variable $x$ (m) and in time $t$ (s). Invariably, a transformation to the frequency domain follows, whereby ordinary differential equations result that are parametric in the frequency $\omega$ (rad/s). The change in cross-section along the length results in dispersion phenomena that would otherwise be absent. Furthermore, it might be that the aforementioned three types of vibration patterns are coupled, but this would result in a problem that is probably intractable in terms of a closed-form-solution. Practically speaking, for a circular cylindrical cross-section only flexural vibrations might be influenced by the presence of an axial force, and this would depend on the external loading configuration and frequency content [8]. In here, we will focus on the axial vibration problem as a first step in exploring the suitability of these models to SHM.

2 MECHANICAL MODEL FOR STRUCTURES OF VARIABLE CROSS-SECTION

Figure 1 depicts a free-standing waveguide placed along the X-axis. For transverse $f(x,t)$ and longitudinal $p(x,t)$ loads distributed along the length of the waveguide, plus initial conditions, a bending moment $M(x,t)$, a shear force $Q(x,t)$ and an axial force $N(x,t)$ develop across the cross-section. Also, $u(x,t)$ and $w(x,t)$ are the axial and transverse displacements, respectively, with $\theta = \partial w / \partial x$ being the slope of the neutral axis. The following definitions for the forces and boundary conditions corresponding to a cantilevered waveguide are given below, with initial conditions assumed to be zero:

$$N = EA(x)(\partial u / \partial x), \quad M = -EI(x)(\partial w^2 / \partial x^2), \quad Q = -EI(x)(\partial w^3 / \partial x^3),$$
$$M(a,t) = Q(a,t) = w(b,t) = w'(b,t) = 0, \quad N(a,t) = u(b,t) = 0$$

(1)
In the above, $EA(x)$ and $EI(x)$ are the aggregate axial and flexural stiffness that varies along the antenna length, $a \leq L \leq b$. Finally, the alternative notation used here involves primes ($'$) and dots ($\cdot$) to respectively denote differentiation with respect to the spatial coordinate $x$ and time $t$. We will consider the following type of smooth variation for the flexural stiffness, the axial stiffness and the distributed mass of the waveguide as

$$EI(x) = (EI)_{base}(x/b)^3, \quad EA(x) = (EA)_{base}(x/b), \quad m(x) = (m)_{base}(x/b)$$  \hspace{1cm} (2)$$

Where $EI$, $EA$, $m$ are the reference values at the base $x = b$, with the antenna length being $L = b - a$. The type of variation assumed dictates the type of the partial differential equation that will result. When the equations of motion are transformed in the frequency domain, they become Bessel and Euler equations for the axial and flexural vibrations, respectively.

\begin{equation}
\frac{\partial}{\partial x} \left[ EA(x) \frac{\partial u(x,t)}{\partial x} \right] - \frac{\partial}{\partial x} \left[ Q(x,t) \frac{\partial w(x,t)}{\partial x} \right] - m(x) \frac{\partial^2 u(x,t)}{\partial t^2} = p(x,t)
\end{equation}

\begin{equation}
\frac{\partial^2}{\partial x^2} \left[ EI(x) \frac{\partial^2 w(x,t)}{\partial x^2} \right] - \frac{\partial}{\partial x} \left[ N(x,t) \frac{\partial w(x,t)}{\partial x} \right] + m(x) \frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t)
\end{equation}

3 EQUATIONS OF DYNAMIC EQUILIBRIUM

By considering force and moment equilibrium of the differential segment $dx$ in Fig. 1, we obtain coupled governing equations of dynamic equilibrium for axial and flexural motion as

3.1 The time domain

If the coupling terms $\partial / \partial x(Q \cdot \partial w / \partial x)$ in the former and $\partial / \partial x(N \cdot \partial w / \partial x)$ in the latter equation are ignored, then by differentiating the variable stiffness terms we recover the following equations:
\[ EA(x) \cdot u'' + (\partial EA(x)/\partial x) \cdot u' - m(x) \cdot \ddot{u} = p(x,t) \]  
(5) 
\[ EI(x) \cdot w''' + 2(\partial EI(x)/\partial x) \cdot w'' + (\partial^2 EI(x)/\partial x^2) \cdot w' + m(x) \cdot \ddot{w} = f(x,t) \]  
(6)

Next, by introducing the variable stiffness representations of Eq. (2) and recover the final form for the equations of dynamic equilibrium as follows:

\[ (EAx/b) \cdot u'' + (EA/b) \cdot u' - (mx/b) \cdot \ddot{u} = p(x,t) \]  
(7) 
\[ (EIx^4/b) \cdot w''' + 8(EIx^3/b) \cdot w'' + 12(EIx^2/b) \cdot w' + (mx/b) \cdot \ddot{w} = f(x,t) \]  
(8)

**Remark 1:** By examining the uncoupling assertions more carefully, we see that omission of the shear force \( Q \) in Eq. (3) implies that \( \partial w^3/\partial x^3 = 0 \), i.e., we recover a polynomial solution in the spatial coordinate for the transverse displacement in the form of \( w(x,t) = g_1(t)x^2 + g_2(t)x + g_3(t) \). A similar state of affairs exists if we neglect the axial force \( N \) in Eq. (4), i.e., we get \( \partial u/\partial x = 0 \) so that \( u(x,t) = (a_1t + a_2)x + (b_1t + b_2) \). Strictly speaking therefore, Eq. (5) and (6) are no longer mathematically consistent with the original system of Eq. (3) and (4). In essence, what we assume is that axial and flexural vibrations in the waveguide are respectively induced by upwards/downwards travelling pressure (P) and horizontally polarized shear (SH) waves, without conversion from one wave type to another.

### 3.2 The frequency domain

For harmonic vibrations, we have the displacements as \( w(x,t) = W(x)e^{i\omega t} \) and \( u(x,t) = U(x)e^{i\omega t} \), where \( e^{i\omega t} = \cos(\omega t) + i\sin(\omega t) \), and likewise for the external distributed forces, i.e., \( f(x,t) = F(x)e^{i\omega t} \), \( p(x,t) = P(x)e^{i\omega t} \). Then,

\[ xU'' + U' + a^2 xU(x) = \tilde{P}(x), \quad a^2 = (m\omega^2) / EA \]  
(9) 
\[ x^4W(x)^{''''} + 8x^3W^{''''} + 12x^2W^{'''} - \beta^4 xW(x) = \tilde{F}(x), \quad \beta^4 = (m\omega^2) / EI \]  
(10)

Note that both \( a(\omega) = \omega/c_p \), \( c_p = (E/\rho)^{1/2} \) (the P-wave speed) and \( \beta(\omega) \) are the usual real-valued wavenumbers associated with vibrations of prismatic beams. Also, \( \tilde{P}, \tilde{F} \) are the scaled spatial variations of the external forces. If the above equations are normalized with respect to the leading derivative terms, we respectively recover a Bessel equation of zero order and an incomplete Euler equation of the fourth order, i.e.,

\[ U'' + (1/x) \cdot U' + a^2 \cdot U = \tilde{P} \]  
(11) 
\[ W(x)^{''''} + (8/x)W^{''''} + (12/x^2)W^{'''} - (\beta^4/x^3)W(x) = \tilde{F} \]  
(12)

**Remark 2:** We note here that a quadratic variation of the mean radius \( D/2 \) of a circular cylindrical pylon of constant thickness \( t \) yields a complete Euler equation of the fourth order. This will be communicated in future work, since the quadratic variation is a more general representation that can replace the model of Eq. (2). For the above flexural vibration case, although the eigenfrequencies can still be recovered from Eq. (12), the key substitution that will give solutions of the type \( W(x) = \sum B_i|x|^m, i = 1...4 \) will not work.

### 3.3 Solution for axial vibrations

The homogeneous solutions to the Eq. (11) is
\[ U(x) = A_1 J_0(ax) + A_2 Y_0(ax) \] (13)

Where \( J_0, Y_0 \) respectively are the Bessel functions of zero order, first and second kind. Next, \( A_i \) are constants of integration to be determined from the boundary conditions. For the tapered waveguide with fixity at the base, we have that \( U(x = b) = 0 \), while for a traction-free condition at the top, we have that \( N(x = a) = 0 \rightarrow U'(x = a) = 0 \). The second boundary condition requires the first derivative of the displacement solution of Eq. (13). This is accomplished by noting that the spatial derivatives of the Bessel functions are given by the following expressions: \( J'_0(ax) = -aJ_1(ax) \), \( Y'_0(ax) = -aY_1(ax) \). Now, for the aforementioned homogeneous conditions, constants \( A_i \) can only be determined relative to each other. This is done using the displacement derivative evaluated at \( x = b \) and setting it equal to zero. Back-substitution into Eq. (13) evaluated at \( x = a \) and subsequently also setting it equal to zero yields the following transcendental equation

\[ J_0(ab) - \{ J_1(aa)/Y_1(aa) \} \cdot Y_0(ab) = 0 \] (14)

An analytical solution for wave number \( a_n = \omega_n/c_p \) values which are the roots of the above equation is not possible, and therefore the Newton-Raphson method is used with starting values being the eigenfrequencies \( \omega_n \) of a reference pylon with constant cross-section. More specifically, if we consider a pylon whose cross-section matches that of the tapered pylon at its base, then \( A = 2\pi db, m = \rho A \). Closed-form solutions for the displacement vector, the wave numbers, the eigenfrequencies and the eigenfunctions are as follows (Graff, 1975):

\[ U(x) = A_1 \sin(ax) + A_2 \cos(ax), \quad a_n = \frac{(2n - 1)\pi}{(2L)} \quad \omega_n = \frac{(2n - 1)\pi c_p}{(2L)}, \quad \phi_n(x) = \sin(a_nx), \quad n = 1, 2, \ldots, \infty \] (15)

Once the eigenfrequencies \( \omega_n \) of the tapered pylon have been determined, then the corresponding eigenfunctions \( \phi_n(x) \) are the displacements given in Eq. (13), after substitution of the corresponding wave number \( a_n \) and with \( A_1 = 1, A_2 = -J_1(aa)/Y_1(aa) \).

3.4 Modal analysis for axial vibrations

Modal analysis commences with the transformation from the physical displacement coordinates \( u \) to the generalized (or modal) coordinates \( q_i \), \( i = 1, 2, 3, \ldots \) by use of the eigenfunctions \( \phi_j \) as follows:

\[ u(x,t) = \sum_{i=1}^{\infty} \left( \phi_j(x) \cdot q_j(t) \right) \] (16)

Substitution of this expansion into the uncoupled form of the equation of motion, Eq. (4a), pre-multiplication by eigenfunction \( \phi_j \) and integration over the length of the waveguide gives

\[ \sum_{j=1}^{\infty} \left( \int_0^L \phi_j(x) \cdot [E A(x) \phi_j'(x)]' dx \cdot q_j(t) - \int_0^L \phi_j(x) \cdot m(x) \cdot \phi_j(x) dx \cdot \ddot{q}_j(t) \right) = \sum_{j=1}^{\infty} \int_0^L \phi_j(x) \cdot p(x,t) dx \] (17)
We note that because of the orthogonality property of the eigenfunctions with respect to the stiffness $EA(x)$ as the weight function, the only non-zero terms remaining are when the two counters coalesce, i.e. $i = j$. This is now followed with integration by parts to shift the spatial derivatives ('') outside the brackets to the eigenfunctions $\varphi_i(x)$. Intermediate terms arising in the integration are zero because of the homogeneous spatial boundary conditions. The final result is a system of single degree-of-freedom (SDOF) equations of the form

$$[M]_i \cdot \{q^i(t)\}_i + [K]_i \cdot \{q(t)\}_i = \{P(t)\}_i$$

where the system matrices involve integration over the waveguide length as

$$[M]_i = \int_0^L m(x) \cdot \varphi_i^2(x) \, dx, \quad [K]_i = \int_0^L EA(x) \cdot \varphi_i''(x)^2 \, dx$$

$$\{P(t)\}_i = \int_0^L \varphi_i(x) \cdot p(x,t) \, dx$$

As with standard modal analysis, the accuracy achieved in solving for the displacement $u(x,t)$ depends on the number of generalized coordinates $q_i(t)$ retained in the series expansion, which from the previous eigenvalue analysis requires no more than four terms. Finally, the integrals over the length of the waveguide are evaluated by low order Gauss quadrature.

## 4 NUMERICAL EXAMPLE

We consider a thin-walled steel pylon used, e.g., in supporting electric cable lines for high-speed trains, see Fig. 1. The pylon cross-section has the shape of a ring with mean radius $R=D/2$ and constant thickness $t$. The length $L = (b - a)$ of the pylon under consideration comprises continuously welded segments for a total length of 12 m, while the base is fixed. Table 1 lists values for the modulus of elasticity $E$, Poisson’s ratio $\nu$ and the mass density $\rho$. At any given station $a \leq x \leq b$, the cross-sectional area is $A = 2 \pi R t$, the principal moment of inertia is $I = \pi R^3 t$ and the mass per unit length is $m = A \rho$. From this information, it is possible to compute both axial $EA(x)$ and flexural $EI(x)$ stiffness, as well as the mass $m(x)$ per length of the pylon. These quantities vary linearly with height for the mass and axial stiffness and cubically for the flexural stiffness, see Eq. (2), with the base of the pylon (btm) considered as the reference point.

<table>
<thead>
<tr>
<th>$E$ (kPa)</th>
<th>$\nu$</th>
<th>$\rho$ (tn/m$^3$)</th>
<th>$a$ (m)</th>
<th>$b$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$21.0 \times 10^7$</td>
<td>0.30</td>
<td>7.85</td>
<td>2.0</td>
<td>14.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$R$ (m)-outer</th>
<th>$t$ (cm)</th>
<th>$A$ (m$^2$)-ref</th>
<th>$I$ (m$^4$)-ref</th>
<th>$m$ (tn/m)-ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.20</td>
<td>2.0</td>
<td>0.025</td>
<td>0.0005</td>
<td>0.197</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$EA$ (kN)-top</th>
<th>$EI$ (kN-m$^2$)-top</th>
<th>$m$ (tn/m)-top</th>
<th>$EA$ (kN)-btm</th>
<th>$EI$ (kN-m$^2$)-btm</th>
<th>$m$ (tn/m)-btm</th>
</tr>
</thead>
<tbody>
<tr>
<td>754,000</td>
<td>15,140</td>
<td>0.025</td>
<td>5,279,000</td>
<td>106,000</td>
<td>0.197</td>
</tr>
</tbody>
</table>

Table 1: Numerical values for the mechanical properties of the steel pylon example.
4.1 Eigenvalue analysis for axial vibrations

As mentioned in sub-section 3.3, we introduce a cantilevered pylon with a constant cross-section whose configuration matches that of the non-homogenous pylon at the base to serve as benchmark for comparison purposes. We now compute its first five eigenfrequencies $\omega_i$ and their respective modal masses $M_i$ and stiffnesses $K_i$, see the left column in Table 2 and also Fig. 3. Similarly, for the tapered pylon, we use the Newton-Raphson method to numerically solve the characteristic polynomial resulting from imposition of the homogeneous boundary conditions to the solution given in Eq. (13). Standard procedure of back-substitution of the recovered eigenfrequencies into the normalized displacement function gives the corresponding eigenfunctions. All these values appear as the right column of Table 2 and in Fig. 3. The computations regarding the tapered pylon were carried out in a Python programming environment [9] requiring negligible running times.

We note that axial vibrations occur at high frequencies, given that the first eigenfrequency of the uniform pylon is 108 Hz and that of the non-uniform one is 146 Hz. Also, the non-uniform pylon consistently has higher values for the eigenfrequencies, because of less mass as compared to the uniform pylon, all other things being equal. Regarding the presence of structural damping in the pylons, it is possible to introduce [10] a complex elasticity modulus in the form $E(1.0+\eta\omega)$, where the structural damping coefficient $\eta$ is in the range of 0.1%. This damping is in addition to the dispersion effects for the tapered pylon and results in complex number formalism for the ensuing numerical solution.

<table>
<thead>
<tr>
<th>Eigenvalue: 1</th>
<th>Eigenvalue: 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_i$: 1.178 tn</td>
<td>$M_i$: 0.336555 tn</td>
</tr>
<tr>
<td>$K_i$: 539743.99 kN/m</td>
<td>$K_i$: 285333.0 kN/m</td>
</tr>
<tr>
<td>$\omega_i$: 677.04 rad/s</td>
<td>$\omega_i$: 920.635 rad/s</td>
</tr>
<tr>
<td>$f_i$: 107.75 Hz</td>
<td>$f_i$: 146.524 Hz</td>
</tr>
<tr>
<td>Eigenvalue: 2</td>
<td>Eigenvalue: 2</td>
</tr>
<tr>
<td>$M_i$: 1.178 tn</td>
<td>$M_i$: 0.156565 tn</td>
</tr>
<tr>
<td>$K_i$: 4857695.92 kN/m</td>
<td>$K_i$: 742137.0 kN/m</td>
</tr>
<tr>
<td>$\omega_i$: 2031.12 rad/s</td>
<td>$\omega_i$: 2184.17 rad/s</td>
</tr>
<tr>
<td>$f_i$: 323.26 Hz</td>
<td>$f_i$: 347.621 Hz</td>
</tr>
<tr>
<td>Eigenvalue: 3</td>
<td>Eigenvalue: 3</td>
</tr>
<tr>
<td>$M_i$: 1.177 tn</td>
<td>$M_i$: 0.16852 tn</td>
</tr>
<tr>
<td>$K_i$: 13493599.77 kN/m</td>
<td>$K_i$: 2.0611e+6 kN/m</td>
</tr>
<tr>
<td>$\omega_i$: 3385.19 rad/s</td>
<td>$\omega_i$: 3493.79 rad/s</td>
</tr>
<tr>
<td>$f_i$: 538.77 Hz</td>
<td>$f_i$: 556.054 Hz</td>
</tr>
<tr>
<td>Eigenvalue: 4</td>
<td>Eigenvalue: 4</td>
</tr>
<tr>
<td>$M_i$: 1.177 tn</td>
<td>$M_i$: 0.634078 tn</td>
</tr>
<tr>
<td>$K_i$: 2444755.54 kN/m</td>
<td>$K_i$: 1.4745e+7 kN/m</td>
</tr>
<tr>
<td>$\omega_i$: 4739.27 rad/s</td>
<td>$\omega_i$: 4822.44 rad/s</td>
</tr>
<tr>
<td>$f_i$: 754.25 Hz</td>
<td>$f_i$: 767.516 Hz</td>
</tr>
<tr>
<td>Eigenvalue: 5</td>
<td>Eigenvalue: 5</td>
</tr>
<tr>
<td>$M_i$: 1.178 tn</td>
<td>$M_i$: 1.61583 tn</td>
</tr>
<tr>
<td>$K_i$: 43719263.25 kN/m</td>
<td>$K_i$: 5.7526e+7 kN/m</td>
</tr>
<tr>
<td>$\omega_i$: 6093.35 rad/s</td>
<td>$\omega_i$: 6160.37 rad/s</td>
</tr>
<tr>
<td>$f_i$: 969.79 Hz</td>
<td>$f_i$: 980.453 Hz</td>
</tr>
</tbody>
</table>

Table 2: Steel pylon with constant cross-section (left) and variable cross-section (right): The first five modal masses, modal stiffnesses and eigenvalues.
4.2 Comparisons with FEM models

The same cantilevered pylon is now modeled using two types of FEM representations, namely a ‘stick’ model comprising 2-noded beam elements with 6 degrees-of-freedoms (DOF) per node and a ‘shell’ model comprising 4-noded shell elements with also 6 DOF per node [11]. The convergence study focuses on the axial vibration eigenvalues and examines the necessary number of finite elements along the length and breadth of the pylon to achieve convergence of the results.

![Figure 3: Steel pylon with (top) constant and (bottom) variable cross-section: The first five normalized eigenfunctions $\phi_i(x)$. Legend: 1 = blue / 2 = red / 3 = green / 4 = yellow / 5 = purple](image)

Starting with the ‘stick’ model, we begin with 10 uniformly spaced elements for the pylon yielding 60 DOF and reach a maximum of 100 elements resulting in 600 DOF. For the ‘shell’ model, the smallest acceptable mesh covers the circumference of the cross-section with 3 shell elements spanning 120° sectors each. Therefore, the comparable numbers for the ‘shell’ model start at 30 elements with 180 DOF and range up to 300 elements with 1800
DOF. Of course, the FEM ‘shell’ model yields vibration patterns that cannot be captured by the waveguide model, such as ‘breathing’ modes where the circumference’s shape diverges from the original circular one. We see in Fig. 4 that roughly 20 finite elements per length are required in most cases before the eigenfrequencies start to converge, yielding a minimum finite element length of 50 cm. This holds true for both ‘stick’ and ‘shell’ models, and we observe that although both FEM models give identical results, they overshoot the analytical waveguide solution by about 2%.

![Image of convergence study graphs](image)

**Figure 4:** FEM convergence study on the first four axial eigenfrequencies for both (a) constant and (b) variable pylon cross-sections. Note: 3 shell finite elements are used to model the pylon circumference.
4.3 Transient axial vibrations

We distinguish two types of quasi-harmonic external loads on pylons supporting electric lines that are generated by the passage of high speed trains: (i) High frequency base accelerations in the vertical direction (axial vibrations) in the range around 50 Hz resulting from the train’s wheels running across rails that have minor imperfections; and (ii) low frequency base accelerations in the horizontal direction (flexural vibrations) in the range around 5 Hz as the train moves rapidly forward. Both cases lead to a forcing function in the form \( p(x,t) = f(x,t) = -mg \sin(\Omega t) \), where \( g = 9.81 \text{ m/s}^2 \) is the acceleration of gravity and \( \Omega \) (rad/s) is the external frequency of vibration. Focusing on axial vibrations, Fig. 5 is a parametric study summarizing the number of eigenfunctions necessary in the modal analysis to achieve convergence of the transient displacement at the top of the pylon, with the time axis ranging as \( 0 < t < 0.4 \text{ s} \). We observe that as the external frequency of vibration increases, so does the number of eigenfunctions necessary for convergence. However, even three eigenfunctions are sufficient, provided the excitation frequency in the axial case does not exceed a value of \( f = \Omega/2\pi = 300 \text{ Hz} \).
5 CONCLUSIONS

An analytical model based on elastic waveguides was presented here for determining the dynamic vibrations of flexible structures such as antennas and pylons to environmentally induced loads. The stiffness of these structures may vary with height and both axial and flexural vibrations were considered, with torsional vibrations to be communicated in future work. These solutions are meant to replace large-scale FEM models for reasons of computational economy within the context of real-time data processing in wireless sensor nodes. This is a crucial step in SHM operations, whereby data streams are transmitted by the sensor nodes to a central processing unit only when calculations show that certain tolerance limits are exceed. These limits can be pre-set, but it is necessary to first quantify and then update them over the course of time. To this end, numerical models for monitored structures, which are both accurate and efficient in terms of computing power, must be introduced.
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